Models of Set Theory II - Winter 2013

Prof. Dr. Peter Koepke, Dr. Philipp Schlicht Problem sheet 1

Problem 1 (4 Points). Suppose that κ is an infinite cardinal. Show that $\mathrm{FA}_\kappa(Q)$ holds for any κ-closed partial order Q.

Problem 2 (4 Points). Show that $\mathrm{FA}_{2^{\aleph_0}}(\mathrm{Fn}(\aleph_0, \aleph_0, \aleph_0))$ is false.

Problem 3 (6 Points). Let $P = \mathrm{Fn}(\aleph_1 \times \omega, 2, \aleph_0)$ and $p \leq q :\Longleftrightarrow p \supseteq q$ for $p, q \in P$. Let G be M-generic on P and $F := \bigcup G$. Let

$$c_\beta \colon \omega \to 2,\ c_\beta(n) = F(\beta, n)$$

for all $\beta < \omega_1$.

(a) Show that in $M[G]$, there is an $\aleph_1$-sequence of measure 0 sets whose union is $\mathbb{R}$.

(b) Show that $\{c_\beta \mid \beta < \omega_1\}$ has measure 0 in $M[G]$.

Problem 4 (6 Points). *Random forcing* $\mathbb{P}$ is defined as the set of Borel subsets p of the real line $\mathbb{R}$ with positive Lebesgue measure $\mu(p) > 0$. Let $p \leq q :\Longleftrightarrow p \subseteq q$. Suppose that M is a ground model and let $\mathbb{P}^M$ denote random forcing defined in M.

(a) Show that $(\mathbb{P}, \leq)$ satisfies the c.c.c.

(b) Suppose that $\epsilon > 0$, $n \in \omega$, and $p \Vdash^M_{\mathbb{P}^M} \dot{f} : \omega \to \omega$. Prove that there are $q \leq p$ in $\mathbb{P}^M$ and $g_n \in \omega$ such that $q \Vdash \dot{f}(n) \leq g_n$ and $\mu(p - q) < \epsilon$.

(c) Let $f \leq^* g \Longleftrightarrow \exists n_0 \forall n \geq n_0 f(n) \leq g(n)$ for $f, g \in {}^\omega\omega$. Show that $\mathbb{P}^M$ is ω^ω-bounding over M, i.e. if G is M-generic on P and $f \in {}^\omega\omega$ is in $M[G]$, then there is some $g \in {}^\omega\omega$ in M with $f \leq^* g$.

Please hand in your solutions on Wednesday, October 23 before the lecture.